

AN EMPIRICAL STUDY OF BOND PORTFOLIO IMMUNIZATION EFFECTIVENESS DURING 2008 GLOBAL FINANCIAL CRISIS

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Abstract

Immunization is a strategy that matches the duration of assets and liabilities to minimize the impact of interest rate changes. This can be achieved using Redington's conditions. This paper addresses a gap in the existing literature: while prior immunization studies typically evaluate performance under stable market conditions or rely on real bond portfolios that confound interest rate risk with credit and liquidity risk, this study isolates pure interest-rate risk by constructing a synthetic default-free bond and liability portfolio priced on historical U.S. Treasury par yield curve data spanning January 2006 to December 2010 — the pre-crisis, crisis, and recovery phases of the GFC. Applying Redington's immunization conditions to this synthetic portfolio, the study finds that the present value of assets equals the present value of liabilities (Test 1: passed) and that asset convexity exceeds liability convexity (Test 3: passed); however, the volatility of the asset cashflows (4.50) diverges substantially from that of the liability cashflows (2.64), so the volatility-matching condition fails (Test 2: failed). Consequently, the portfolio is only partially immunized: it remains exposed to small parallel shifts in the yield curve despite its favourable convexity position for larger shifts. The findings indicate that duration-based immunization, even when correctly specified for present-value matching, requires explicit volatility matching to provide reliable protection during periods of extreme interest rate volatility such as the 2008 GFC, and that partial immunization can still leave institutional investors exposed to material losses.

Keywords

Bond Portfolio, Immunization, Global Financial Crisis, Financial Volatility, Crisis 2008

1. Introduction

As an example, for immunization imagine an organisation has liabilities that will require a known series of cashflows (which we will assume are all negative) and holds assets that will generate a known series of cashflows (which we will assume are all positive) to meet these liabilities. If it were possible to select a portfolio of assets that generate cashflows that exactly matched the liabilities of the fund (in terms of timing and amount), then the fund

would be completely protected against any changes in interest rate. However, this is an idealised scenario and, apart from in very simple cases, perfect matching of this kind cannot be achieved. Suppose the present value of the fund's liabilities and assets, calculated at a valuation rate of interest i , which reflects the interest rate in market, are $VL(i)$ and $VA(i)$, respectively. Then the fund has a surplus of $S(i) = VL(i) - VA(i)$. We can consider how this surplus

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would be affected by changes in interest rate i . In particular, we would be concerned about the downside risk if a change in market interest rate causes the surplus to become negative, i.e. a deficit.

In the 1950s, actuary Frank Redington derived the three conditions that are required to achieved immunisation. The conditions for Redington's immunisation maybe summarised as follows:

1. $VA(i_0) = VL(i_0)$ – that is, the value of assets at the starting rate of interest is equal to the value of liabilities.
2. The volatilities of the asset and liability cash-flow series are equal, that is, $vA(i_0) = vL(i_0)$.
3. The convexity of the asset cashflow is greater than that of the liability cashflow series – that is, $cA(i_0) > cL(i_0)$.

In words, a fund is said to be immunized against small changes in interest rate if:

1. the surplus in the fund at the current interest rate is zero and
2. any small changes in the interest rate (in either direction) would lead to a positive surplus.

1.1 The Global Financial Crisis 2008

The 2008 Global Financial Crisis (GFC) stands as a critical stress test for financial markets, exposing the limitations of conventional investment and risk management practices. For fixed-income investors, the extreme volatility in interest rates and severe liquidity shortages raised important questions about the resilience of established portfolio protection strategies. Bond portfolio immunization, a duration-based approach designed to safeguard portfolio returns against interest rate fluctuations, has long been regarded as a theoretically sound method of risk control. However, its real-world effectiveness during systemic crises remains contested. This study undertakes an empirical analysis of immunization performance during the GFC, evaluating its capacity to preserve portfolio value amidst parallel

and non-parallel shifts in the yield curve, heightened credit risk, and unprecedented market dislocations.

2. Review of Literature

Kałużska (2004) studied on "On risk minimizing strategies for default-free bond portfolio immunization." Traditional bond portfolio immunization strategies, such as those based on duration matching (e.g., Macaulay and Fisher-Weil duration), aim to protect portfolios against small parallel shifts in interest rates. Building on this, Kałużska and Kondratiuk-Janyska (2004) extend immunization strategies by modeling interest rate shocks as random variables with known distributions, enhancing realism. They propose risk-minimizing strategies that incorporate both stochastic modeling and liquidity considerations, thus contributing to the growing literature on robust fixed-income risk management.

Rudd, Kevin (2009) examined on "The global financial crisis." offers a critical perspective, arguing that the crisis exposed fundamental flaws in unregulated free-market capitalism, particularly the belief in self-correcting markets. His analysis draws on earlier critiques of market fundamentalism, echoing concerns raised by economists like Joseph Stiglitz and Paul Krugman, who emphasized the need for greater regulatory oversight and active government intervention. Rudd contributes to this discourse by advocating for a social-democratic response to the crisis, emphasizing fairness, public investment, and rebalancing market and state roles. Rudd's article stands out by combining economic analysis with political ideology, reflecting a policymaker's perspective during a time of systemic uncertainty.

Wang (2024), "Analysis of Bond Immunization Strategies Based on Duration Model." This study examines duration-matching immunization strategies with an emphasis on maturity immunity of bond portfolios, analysing the matching of maturity

and yield with the investment cycle. The authors identify limitations in maturity-based immunization under fluctuating interest rate conditions and offer recommendations for investors to maintain income stability and mitigate interest-rate-driven losses, reinforcing the relevance of duration and convexity as complementary — not substitute — risk measures.

Chua (2018) stated that, "Incorporating Convexity in Bond Portfolio Immunization Using a Multifactor Model: A Semidefinite Programming Approach." This paper extends single-factor duration immunization by incorporating second-order convexity effects into a multifactor framework, reformulating the immunization problem as a semidefinite program. Simulation and empirical results demonstrate that second-order (convexity-inclusive) immunization strategies more accurately approximate bond value changes and outperform first-order duration-only strategies, supporting the theoretical importance of the convexity condition tested in this paper.

Morau (2024) studied, "The Mean–Variance (In)Efficiency of Duration-Based Immunization." Using U.S. yield curve data from 1977–2020, the authors show that duration-based immunization is not minimum-variance and can become temporarily inefficient, particularly under non-parallel yield curve shifts. Their mean-variance framework helps explain why simple duration-matching portfolios sometimes underperform more sophisticated strategies, offering theoretical grounding for cases — such as this paper's findings — where present-value matching alone (without volatility matching) fails to deliver full immunization.

3. Research Gap

Existing literature on bond immunization largely falls into two categories: theoretical extensions of Redington's model under stochastic interest rate assumptions (e.g., Kałuska & Kondratiuk-Janyska,

2004; He & Wang, 2024), and empirical tests conducted during ordinary, non-crisis market conditions using real-world bond portfolios, which necessarily conflate interest rate risk with credit and liquidity risk. Few studies isolate the pure interest-rate-risk performance of Redington immunization specifically across the 2008 GFC window using a controlled, default-free instrument. This paper addresses that gap by constructing a synthetic bond and liability portfolio, free of credit and liquidity effects, and testing it against all three Redington conditions using actual U.S. Treasury par yield data from 2006–2010. In doing so, it provides direct empirical evidence on whether classical immunization theory held up under one of the most severe interest rate stress events in recent history.

4. Methodology

This paper adopts a quantitative, empirical approach to evaluate the effectiveness of bond portfolio immunization during the 2008 Global Financial Crisis and the subsequent recovery period. The analysis is conducted using synthetic fixed-income securities priced with historical Treasury yield curve data sourced from the U.S. Department of the Treasury. The use of synthetic instruments ensure data consistency, eliminates liquidity biases, and allows precise control over maturity, coupon rate, and payment structure.

The 5-year, 4% annual-coupon structure was selected as a stylized representation of a medium-duration institutional bond holding, consistent with typical insurance and pension fund exposures used in Redington-style immunization studies. The two hypothetical liabilities (\$57 at the end of 2007 and \$50 at the end of 2009) were constructed so that their combined present value approximates that of the asset under the prevailing yield curve, allowing the first Redington condition (present value match-

ing) to be satisfied at the outset — a standard starting point in the immunization literature before testing whether the remaining two conditions (volatility and convexity matching) also hold. U.S. Treasury par yields were used specifically because they represent the risk-free benchmark rate against which credit and liquidity spread effects can be cleanly excluded.

It is acknowledged that this analysis tests a single bond-liability configuration rather than a range of maturities or coupon structures; this is a deliberate simplification appropriate for an initial, controlled test of Redington's conditions during the GFC, and represents a direction for further robustness testing in future research.

The paper then calculates and compares the present value, volatility, and convexity of the asset and liability cashflow series in order to test each of Redington's three immunization conditions in turn. This comparison enables a direct assessment of the extent to which classical immunization theory held during the 2008 Global Financial Crisis.

Pricing Basis: Historical U.S. Treasury nominal par yield curve rates for corresponding maturities, obtained from the U.S. Department of the Treasury
Study period: January 2006 to December 2010, covering the pre-crisis, crisis (2008), and post-crisis recovery phases

5. Analysis and Discussion

5.1 Synthetic Bond and Liability Construction

A hypothetical (synthetic) fixed-income security is constructed to facilitate controlled analysis of portfolio immunization effectiveness. The synthetic bond replicates the cashflow structure of a real coupon-bearing Treasury security but excludes credit risk and liquidity effects. For this paper, the bond is defined as follow:

Face value: \$100

Coupon rate: 4% annually in arrears (paid once per

year)

Payment frequency: Annual coupon and principal repayment at maturity

This synthetic bond serves as the base instrument for constructing both immunized and non-immunized portfolios. The controlled design enables precise evaluation of interest rate risk management strategies without interference from market-specific anomalies.

Consider a hypothetical liability happening in the same time period of \$57 at the end of the year 2007 and another liability of \$50 at end of the year 2009.

Table 1. Interest rates during the period 2006 to 2010

<i>Date</i>	<i>Interest rate for 5yr</i>
29/12/2006	4.70
31/12/2007	3.45
31/12/2008	1.55
31/12/2009	2.69
31/12/2010	2.10

Source: U.S. Department of the Treasury
 (<https://home.treasury.gov/>)

5.2 Immunization Tests

Test 1: Present Values of Asset and Liability

Calculating the price of the bond (asset)

$$\text{Price (present value)} = 4(1.047)^{-1} + 4(1.0354)^{-2} + 4(1.0155)^{-3} + 4(1.0269)^{-4} + 104(1.021)^{-5} = 109$$

Calculating the Yield to Maturity

$$108.7104 = 4(1+i)^{-1} + 4(1+i)^{-2} + 4(1+i)^{-3} + 4(1+i)^{-4} + 104(1+i)^{-5}$$

$$i = 1.021436 \text{ or } 2.1436\%$$

Test 1: Present Value of Asset and Liability

$$\text{Present value of asset} = 4a_{\overline{5}|@2.1436\%} + 100(1.021)^{-5} = 109$$

The Present value of liability = $57(1.021436)^{-2} + 50(1.021436)^{-4}$
 = 109

PV of Assets = PV of Liability

Therefore, Test:1 Passed

Test 2: Volatility of Asset and Liability

Volatility of Asset: = $\{-[(-1)(4)(1.02136)^{-2} + (-2)(4)(1.02136)^{-3} + (-3)(4)(1.02136)^{-4} + (-4)(4)(1.02136)^{-5} + (-5)(104)(1.0236)^{-6}]\} / \text{PV of Asset}$
 = 4.496689

Volatility of liabilities = $\{-[(-2)(57)(1.02136)^{-3} + (-4)(50)(1.02136)^{-5}]\} / \text{PV of Liability}$
 = 2.639486

Volatility of Assets $\neq$ Volatility of Liability Therefore, Test: 2 Failed

Test 3: Convexity of Asset and Liability

Convexity of Asset: = $[(-1)(-2)(4)(1.02136)^{-3} + (-2)(4)(-3)(1.02136)^{-4} + (-3)(-4)(4)(1.02136)^{-5} + (-4)(-5)(4)(1.02136)^{-6} + (-5)(-6)(104)(1.0236)^{-7}] / \text{PV of Asset}$
 = 25.69397

Convexity of Liabilities = $[(-2)(-3)(57)(1.02136)^{-4} + (-4)(-5)(50)(1.02136)^{-6}] / \text{PV of Liability}$
 = 10.99413

Convexity of Assets > Convexity of Liability
 Therefore, Test 3: Passed

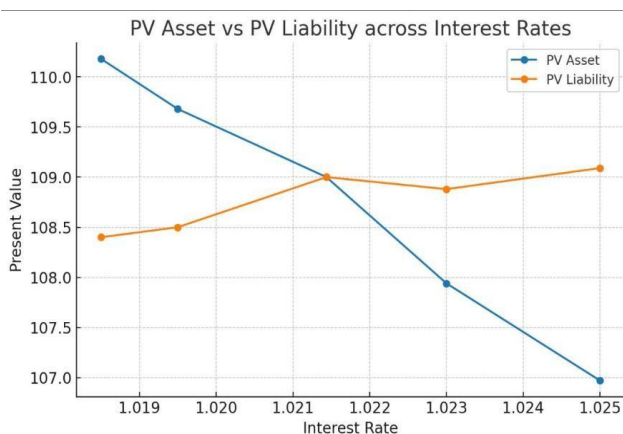


Figure 1. Present value of assets and liabilities across interest rates

The large vertical gap between the PV Asset and

PV Liability lines at interest rates away from the matching point means:

- Mismatch risk increases as interest rates move from the central rate — the asset value falls faster than the liability value when rates rise, and rises faster when rates drop.
- This widening gap is a visual sign of the failed volatility test — the slopes are not matched, so even small rate changes lead to bigger asset–liability mismatches.
- In practical terms, the portfolio is more sensitive to rate shifts than the liabilities, so it will over-react compared to the liability side.

The portfolio is not immunized against small changes in interest rates. Invest in this bond at this rate of interest with these liabilities is not recommendable because a small change in interest rate will lead to mismatch between assets and liabilities.

6. Conclusion

Although the portfolio has equal present value to the liabilities and enjoys a higher convexity, the mismatch in the volatility means it does not satisfy the complete Redington immunization conditions. The portfolio is partially immunized – it will still be exposed to interest rate risk from small parallel shifts in yields, even though it may perform relatively better for larger shifts due to higher convexity.

If a portfolio like this for example is not being immunized during that period, the investor/ the company would have definitely faced losses.

In a situation like The Global Financial Crisis 2008, immunization from interest rate swings, ensures liabilities are met without facing losses and maintains stability without forced asset sales despite market turmoil.

This finding is consistent with the broader literature (François & Moraux, 2024; Chua, 2018), which shows that present-value matching alone does not

guarantee protection against interest rate risk unless the volatility (and, where relevant, higher-order convexity) of asset and liability cashflows are also matched. Future research could extend this analysis by testing a wider range of bond maturities and coupon structures, and by incorporating non-parallel yield curve shifts, to assess the robustness of these findings beyond the single portfolio configuration examined here.

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